Vermont Elementary and Middle Level Mathematics Problem Solving Assessment Guide

This scoring guide was developed by
the Vermont Department of Education
in collaboration with the
Vermont Institute for Science, Math and Technology.

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Problem Solving Criteria

Approach and Reasoning

START HERE

Level 1	Level 2	Level 3	Level 4
• Approach does not work **or** • No approach evident	• Approach would[1] lead to solving only part of the problem[2] or reaching a partial solution **or** • Approach would work but there is some flaw in the reasoning	• Approach works or would[1] work for solving the problem, and reasoning, if evident, is not flawed (Note: Use of a known formula is an approach that works or would work)	Approach works, and at least one of the following three additional aspects of good problem solving is evident. • **Justifies the application of a known formula or rule** used to solve all or part of the problem **or** • **Makes and uses a formula or rule** to solve all or part of the problem **or** • **Describes verification** of her/his solution[3] Shows confidence that the answer is correct by using more than one process and compares the answers to solve the same problem.

Connections

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Level 1	Level 2	Level 3	Level 4
• Response stops without including a mathematically relevant observation with respect to her/his solution	• Makes a mathematically relevant **observation** about her/his solution **or** • Identifies an **underlying** mathematical **concept or pattern** in her/his solution **or** • Solves the problem and then **recreates**[4] the problem and finds a new solution **or** • Solves the problem and then uses a **different** mathematical **process** to solve the same problem	• Relates this problem to a similar problem **by expressing** the **mathematical relationship(s)** **or** • **Analyzes the relationship** among elements in her/his solution or among similar or different mathematical topics in her/his solution **or** • **Tests** and accepts and/or rejects an **hypothesis or conjecture** about her/his solution **or** • Identifies a formula or rule, while solving the problem, that works or would work in solving all or part of **that** problem.	• Solves the problem, discovers a general rule[5] about the solution[3], and **demonstrates understanding of the generalization** either through explanation of the derivation, or through application to at least two other cases **or** • Solves the problem, and then **extends** her/his solution to more **complicated mathematics** **or** • **Evaluates** the **reasonableness** or **significance** of her/his solution

Solution

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Level 1	Level 2	Level 3
• No work is present **or** • No part of the solution[3] is correct **or** • Some work is present, but the work doesn't support the answer given	• The solution[3] is correct for only part of the problem, and there is work to support these correct part(s) **or** • The solution[3] contains mathematical errors which leads to an incomplete or incorrect answer	• The answer is correct, and the work in the solution[3] supports the answer. The answer is identifiable but not necessarily indicated (circled, underlined, stated, etc)

1 **Would:** An approach that would work for solving the problem addresses all aspects of the mathematical situation presented in the task and could lead to a correct solution.

2 **Part of the Problem:** Within a problem, there may be several mathematical components that need to be addressed, or there may be multi-parts. If not all of the mathematical components of the problem are addressed, or not all of the parts of the problem are addressed, then the student only found an approach to solve part of the problem.

3 **Solution:** All of the work that was done to solve the problem.

4 **Recreates:** The student substitutes different numbers in the same problem and finds another solution, or uses the same procedure in a different circumstance.

5 **General Rule:** A rule that can be used no matter what the numbers in the problem are, either expressed in algebraic notation or in words.

Communication Criteria

Mathematical Language: Terms/vocabulary and symbolic notation

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Level 1	Level 2	Level 3
• Is absent **or** • Contains significant flaws in accuracy **or** • Is limited to the language of computation vocabulary and notation **or** • Is limited to formulas that appear without explanation, derivation, or use	• Is **relevant**, but may contain minor flaws **and** • Uses at least two non-computational language terms **from the task** and/or terms brought into the solution: ◦ Number sense and numeration, number relationships, number systems, and number theory (including fractions and decimals) **or** ◦ Geometry and measurement[6], **or** ◦ Statistics and probability, **or** ◦ Patterns, functions, and algebra	• **Uses a variety** of accurate, relevant, non-computational language which goes beyond that presented in the task including the language of… ◦ Number sense and numeration, number relationships, number systems, and number theory (including fractions and decimals) **or** ◦ Geometry and measurement[6] **or** ◦ Statistics and probability, **or** ◦ Patterns, function, and algebra **OR** • Use of algebraic or other notation(s)[7]

Mathematical Representation: Graphs, plots, charts, tables, models and diagrams

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Level 1	Level 2	Level 3
• **Makes no** attempt to make any mathematical representations to solve or communicate an aspect of her/his solution, regardless of the correctness of the solution **or** • Makes only **inappropriate** mathematical representations to solve or communicate an aspect of her/his solution regardless of the correctness of the solution	• **Attempts** to make an appropriate mathematical representation to solve or communicate an aspect of her/his solution, regardless of the correctness of the solution, but the representation lacks labels and/or accuracy with regard to the student's solution. (Note: Completion of a teacher structured representation cannot earn above a level 2)	• Makes an **appropriate** and **accurate** mathematical representation to solve or communicate an aspect of her/his solution, regardless of the correctness of the solution. See glossary for requirements. (Note: The student's text may supply the necessary labeling).

Documentation

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Level 1	Level 2	Level 3
• The documentation of the student's correct or incorrect solution contains **little or no evidence** of how the problem was solved or the reasoning used	• The documentation of the student's correct or incorrect solution **contains some clear parts**, but there are gaps in how the student solved the problem or the reasoning used.	• The documentation of the student's correct or incorrect solution **clearly shows the answer, how the problem is solved, and the reasoning used.** This may be evident by some of the following: ◦ Results of any necessary computation are present ◦ Presentation is in logical order ◦ Representation(s) are linked to text ◦ All parts are connected and labeled

[6] **Measurements:** Attributes of length, capacity, weight, mass, area, volume, time, temperature, and angle

[7] **Notation:** Includes the use of algebraic equations and formulas (with all variables defined), and/or other notations (',", !, ∑, and exponential notations)

[8] **Accurate:** Mathematical representations that are technically correct and executed properly, including labels. See over.

Mathematical Representation Glossary

In order to ensure that student responses receive a "Level 3" on the portfolio scoring criteria for Mathematical Representations, they should reflect the descriptors stated below. Note: On all representations, labeling may serve as a title. The student's text may also supply the necessary labeling.

GRAPH: A diagram showing a relationship between 2 variables.
A graph should be accurate, appropriate to the task, and have correctly scaled and labeled axes or sectors, with data accurately recorded.

PLOT: Stem and Leaf Plot, Scatter Plot, Line Plot, and Box and Whisker Plot.
A plot should be accurate, appropriate to the task, and have any necessary keys and/or scales. Note: Data used in creating a plot should be clearly stated and/or listed within the text.

CHART: Information displayed in rows and columns with no particular order.
A chart should be accurate, appropriate to the task, have labeled rows and columns, and any necessary keys.

TABLE: A systematically ordered chart.
A table should be accurate, appropriate to the task, have labeled rows and columns and any necessary keys.

Special Case:

Systematic List: A list of information that is organized systematically

- Level 3: Is systematically organized, accurate, appropriate, labeled, and has any necessary keys
- Level 2: Is organized and appropriate, but lacks accuracy, labels, and/or necessary keys
- Level 1: A labeled list lacking systematic organization

DIAGRAM: An explanatory drawing.
A diagram should be appropriate to the task, explanatory in nature, have labels, and any necessary keys.

MODEL: A representational drawing or construction; such as sets of plans, scale representations or structural designs.
A model should be accurate, appropriate to the task, and have any necessary keys.